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SURFACE EFFECTS FROM AN UNDERWATER EXPLOSION (REVIEW)

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INTRODUCTION

The surface effects accompanying an underwater explosion represent an integral combination of many interconnected phenomena. Among these we include the special characteristics of the wave field structure, which is determined by the existence of regions of regular and irregular shock-wave reflection from the free surface with sharply differing structure and parameters, the development of cavitation and the formation of a cupola, vertical and radially directed eruptions, which can originate almost simultaneously or alternately, and surface waves. The significant change of shape of the cavity with the detonation products pulsating in the vicinity of the free surface exerts an effect on the nature of the development of the surface eruptions and on the secondary pressure field in the liquid.

As an independent trend in the field of explosion hydrodynamics, investigations of the surface effects from underwater explosions already have a 60-year history. Two of the most fruitful stages of their development may be mentioned: foreign investigations of the 1940's [1, 2] and Soviet investigations in the 1950-1960's [3-10]. One of the first papers in this field is the report of Hillar (1919), mentioned in [2] and devoted to the investigation of the mechanics and parameters of cupolar formation at the surface of a liquid. In this paper an analogy is drawn between the mechanics of cupola formation and the Hopkinson effect (1914), and the suggestion is made that the initial velocity of its ascent for relatively weak shock waves should be characterized by twice the post-shock particle velocity. It is well known that this hypothesis was largely verified by numerous later experiments. Anomalies have been recorded only for the case of the explosion of large-scale charges [1] and have not been explained up to the present.

In [1], the results of investigations of mainly large-scale explosions of charges with weights up to 450 kg were generalized and the relationship between the dynamics of the explosion cavity with the formation of plumes and the characteristics of the parameters of the secondary pulsations was established. The surface effects from underwater nuclear explosions are discussed in [11]. Reference [2], which was issued almost simultaneously with [1], should be specially mentioned, since it is the first and (up to recent times) only attempts to give an analysis of the mechanics and structure of the throwouts at the free surface. This paper is not widely enough known among the circle of specialists in the field.

The first results of a theoretical analysis of the nonlinear interaction of shock waves with the stress relief waves from the free surface were published in [3], which investigated the structure and parameters of shock waves and determined the boundary of the region of irregular reflection. In [4] this problem was studied experimentally, in [5] the solution of the two-dimensional problem concerning the reflection of a spherical shock wave from a free surface was obtained, and in [6] a number of approximate analytical relations, connected with estimates of the parameters of the wave field, were given.

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Detailed experimental investigations and estimates of the parameters of the cupola, plume, and shock waves in the atmosphere, generated by the high-velocity cupola, have been conducted in [7-10, 12]. We note that in [8] the possibility of modeling the surface effects from large-scale explosions by means of small charges under laboratory conditions was shown.

Finally, mention should be made of review [13], which presented a summary of the principal results of most work on underwater explosions occurring after the appearance of monograph [1].

The experimental data obtained are very voluminous, but, nevertheless, a number of the major problems concerning the nature of surface phenomena have remained unsolved. In particular, the structure and mechanics of formation of directional plumes and the inconsistencies between the calculated and experimental data on the parameters of the stress relief waves remain unexplained to a large degree. Because of this, at the end of the 1960's, the Institute of Hydrodynamics Siberian Branch, Academy of Sciences of the USSR, under the initiative of Academician M. A. Lavnevok, conducted investigations directed toward finding new formulations for the purpose of setting up mathematical models of surface phenomena and verifying them experimentally. We note that the first mathematical models of the development of a plume are attributed to Lavrent'ev and Shabat [14] and Ovsyannikov [15].

The review of the principal results given below follows the sequence of development of the phenomena and is compiled in such a way that the completeness is not impaired and detailed repetition of those results, which are partially given in [13] is avoided.

1. Cavitation, Dynamic Strength of a Liquid to a Shock Wave

We shall consider the structure of the wave field and the process of cavitation development as a result of reflection from the free surface of a shock wave from an underwater explosion. The region of the effect of the rarefaction waves on the structure and parameters of the shock wave can be divided into three zones [6]. The first is the zone of irregular reflection (or nonlinear interaction), the essence of which consists in that the rarefaction waves from the free surface, propagating behind the shock front with velocity c + u (far zone or plane wave), overtake and attenuate it. The amplitude and positive phase of the wave in this zone, depending on the location of the recording point, may be reduced considerably; the pressure in the "tail" part of the wave diagram falls slowly [4]. The second is the intermediate zone, for which only the latter effect is valid and the amplitude of the wave is maintained. The third is the zone of regular reflection, for which the wave profiles with a sharply terminating trailing edge in the region of negative pressures are characteristic, intense tensile stresses are recorded.

It has been established by experimental investigations that it is precisely in the zone of regular reflection that cavitation occurs; according to the definition in [16], the cavitation in this zone is assumed to fully be developed (or visible) if the cavitation nuclei in it have attained or have exceeded a diameter of 10^{-2} cm. Consequently, the lower boundary of the zone of irregular reflection, described by the trajectory of the point at which the fronts of the attenuated shock wave, the incident wave and the rarefaction wave converge from a point of the free surface on the axis of symmetry, can be considered as the potentially possible upper boundary of the region of cavitation. For example, in the case of explosion of a 1-9 charge of radius $a_0 = 0.53$ cm at a depth of h = 3 cm, the coordinates of this boundary point x and y have the following values (in cm): (8.5; 0), (9.5; 0.11), (10.6; 0.22), and (11.7; 0.33) [17], where the value of x was read off along the free surface and that of y was read off from the free surface into the depth of the liquid. The experimental data for the corresponding values of x are markedly low, but those for y are high.

Thus, in order to investigate the dynamics of the development of cavitation, it is necessary, first of all, to know the parameters of the rarefaction wave in the zone of regular reflection and, secondly, to have data with respect to the presence of cavitation centers. The former are determined simply on the basis of the principle of superposition of the positive and negative waves (from the explosion of an imaginary charge), taking account of the appropriate delay times. For this, consideration should be given to the fact that these parameters determine only the initial prerequisite of cavitation development. This approach was used, for example, in [18] for calculating the maximum amplitudes of the negative pressures in the case of an explosion, near the free surface, of charges with a weight up to 5 kg at depths of up to 12 m. Gas bubbles or solid inclusions are usually considered as cavitation centers.

In addition, it is necessary to have a suitable mathematical model, which will allow the process to be studied subject to the existence of the data given above. Until recently, there has been no such model. The problem usually has been considered within the framework of a one-phase medium, and it has been assumed that there are certain critical values of the tensile stresses which the liquid does not withstand. Naturally, in this case the process of cavitation development as such has not been investigated. It has been assumed [6, 19] that cavitation develops at the surface of the critical isobar and that spalling of a layer of homogeneous liquid takes place over that surface, i.e., the critical value of the tension at which development of visible cavitation is observed determines the dynamic tensile strength of the liquid.

The existing theoretical and experimental data concerning the critical stress values are clearly of a contradictory nature. For example, under static conditions, the lower and upper values from published estimates differ by three orders of magnitude. According to the results of dynamic experiments, the following data have been obtained [20]: Knapp — not greater than 3.51 atm; Davis et al., — 14 atm; and Brown — 20-37 atm. These are the values of the tensile stresses at which, in the opinion of the authors, cavitation starts to develop. It is fixed from the instant of appearance of visible bubbles, and the magnitude of the stress relief is determined by the readings of indicators (or some other method) at this same instant. It can be seen that the spread of the data is large, which leads to indeterminacy of the problem posed concerning the location and possibility of the development of cavitation (and spallations) within the scope of the above approach. But the contradiction does not end here: The maximum amplitude of the rarefaction wave, recorded in the zone of regular reflection in experiments on an underwater explosion, differs significantly (sometimes by orders of magnitude) from that calculated by the principle of superposition in the region of cavitation development (and even in the region where cavitation is not observed).

Analysis of the well-known papers showed that the root of the contradictions is concealed in the representation of the real liquid by a one-phase medium. At first sight, this approach is entirely natural. According to statistical experimental data (for example, [21]), the content of free gas in the actual liquid is extremely small. It is distributed in the form of cavitation nucleating centers, the size of which depends on the state of the liquid and varies over the range $R_0 \approx 10^{-2}$–10^{-5} cm; the volume concentration of the gas is $k_0 \approx 10^{-8}$–10^{-12}. The minimum values of these parameters are characteristic, for example, of distilled water. Therefore, it is no wonder that the shock waves are not "sensitive" to this structure of the medium. However, there is no basis for assuming that this must also be true with respect to the rarefaction waves. It may be supposed that an increase of these cavitation nucleation centers in the rarefaction wave field leads to the intense development of cavitation, i.e., to a considerable change of state of the medium and, consequently, also of the parameters of the wave field in it.

Thus, we arrive at the necessity of posing the problem concerning the development of cavitation near the free surface within the framework of a model of a two-phase medium, which was postulated and investigated in [17, 22]. The motion of such a medium is described by the Jordan–Kogarko Jr. system of equations, the special feature of which is the complex form of describing the state of the medium, including a second-order nonlinear equation (of the Lamb–Rayleigh type) for a pulsating gas bubble. The average pressure in the medium and the pressure in the gas bubbles do not coincide. When analyzing this system, the following two simplifying factors are frequently used: The compressibility of the liquid–gas bubble medium is determined mainly by the compressibility of the gas phase (i.e., the liquid component of the medium can be assumed to be incompressible) while the nonlinearity of the process is mainly determined by the dynamics of the gas bubble. The latter presents the possibility of limiting the consideration to a linearized system, which in the case of two space variables has the form

$$\rho_t + u_r + r^{-1}v_\theta + 2r^{-1}u + r^{-1}v\,\mathrm{ctg}\,\theta = 0,$$
$$u_t + p_r/3 = 0,\ v_t + p_\theta/3r = 0,$$
$$\rho = (1 + k_0k)^{-1},\ k_{tt} = -k^{1/3}(p - k^{-\gamma}),$$

where $u = u'\sqrt{\rho_0/3p_0}$; $v = v'\sqrt{\rho_0/3p_0}$; $t = t'\sqrt{3p_0/\rho_0R_0^2}$; $\rho = \rho'/\rho_0$; $p = p'/p_0$; $r = r'/R_0$; $k = (R/R_0)^3$. Here ρ, u, v, and p are the averaged dimensionless density and components of the velocity and the pressure; R_0 and k_0 are the initial radius of the bubble and the volume concentration of gas; γ is the adiabatic index of the gas in the bubbles; and r and θ are the coordinates of a spherical system. In considering the problem of cavitation development near the free surface in the case of an underwater explosion, we retain the principle of superposition in order to determine the pressure in the medium. However, in contrast to the one-phase model, the form of the function p(t) will now be determined not only by the change in pressure in the expanding cavity with the detonation products, but also by the change in the volume concentration of the free gas k(t). The determination of this relation, based on an analysis of system (1.1), permits the numerical investigations to be simplified considerably, since together with the pulsation equation it comprises a closed system describing the process being investigated.

With certain additional assumptions, the function p(k) is obtained in [17]:

$$p = k^{-\gamma} + r^{-1/2} \sum_{n=0}^{\infty} A_n K_{n+1/2}(r) P_n(\cos\theta), \tag{1.2}$$

where $r = r'\sqrt{3k_0k^{1/3}}/R_0$; A_n is a constant; and $K_{n+1/2}$ and P_n are a modified Bessel function and a Legendre function, respectively. Now the problem of the development of the cavitation zone and the profile of the rarefaction wave in it can be formulated.

In an unbounded liquid containing uniformly distributed cavitation nucleating centers of radius R_0 and having a volume gas concentration k_0, there are two cavities of initial radius a_0 with the detonation products located at a distance h from one another. Both cavities are expanding according to an adiabatic law with a known value of γ_1, and the initial pressures p(0) in them are known, are equal, and differ in sign. Thus, the pressure at an arbitrary point of the medium is determined by the superposition of solutions of the type (1.2), taking account of the phase shift σ, simulating the delay of arrival of the pressure wave from the explosion of the virtual charge at the point with coordinates r, θ. The constants in the expression for the pressure are determined from the boundary conditions at the cavities with the detonation products. If the condition $a \ll h$ is satisfied for the running radius of the explosion cavity $a(t)$, then the expressions for the constants are simplified and the problem reduces to the solution of a simple system with the corresponding initial conditions [17]

$$\begin{gathered}(p-k^{-\gamma})/p(0) = -\frac{a^{-3\gamma_1+1}}{r}\sigma e^{-\alpha(r-a)} + \frac{a_1^{-3\gamma_1+1}}{r_1}e^{-\alpha(r_1-a_1)},\\ \frac{d^2k}{dt^2} = -k^{1/3}(p-k^{-\gamma}) + \left(\frac{dk}{dt}\right)^2/6k,\end{gathered} \tag{1.3}$$

where a and r are taken relative to a_0 — the radius of the explosive charge; $\alpha = \sqrt{3k_0k^{1/3}}\cdot a_0/R_0$; and the subscript 1 refers to the real charge and to the coordinate of the point in the system with center in the real charge.

Numerical investigations of system (1.3) showed that, for example, in the case of the explosion, close to the free surface, of a 1-g charge at a depth of 5.3 cm in water with initial parameters of the gas content $R_0 = 5\cdot10^{-5}$ cm and $k_0 = 10^{-11}$, there develops a cavitation zone in which the value of k is increased by 7-9 orders of magnitude by comparison with k_0. By remaining within the framework of the determination [16] concerning the cavitation developed, it is simple, for every fixed moment in time t, to isolate the corresponding zone from the whole region (Fig. 1a, $t = 48\cdot10^{-6}$ sec). Comparison with the experimental data (Fig. 1b, the same moment in time) indicates their satisfactory coincidence. Here 1 is the free surface, 2 is the cavity with the detonation products, 3 is the cavitation zone, and $h = 10a_0$. The calculation allows the "lifetime" of the cavitation bubbles to be determined at an arbitrary point, i.e., the interval of time during which it has a size $\geq 10^{-2}$ cm. Its disappearance from the field of view does not mean the absence of cavitation, but indicates only the decrease of the volume concentration of gas in it by comparison with the quantity 10^7k_0.

In [22], within the scope of this formulation, a similar investigation of the parameters of the rarefaction waves in a real liquid was carried out. It was found that the maximum negative pressures recorded in the medium depend not so much on the amplitude of the tensile stresses as on their time of application. Figure 2 shows an example of the calculation of the parameters of a rarefaction wave under the conditions given above at a point with the coordinates $r' = 3.8$ cm and $\theta = 0$. It is found that if the maximum of the rarefaction wave

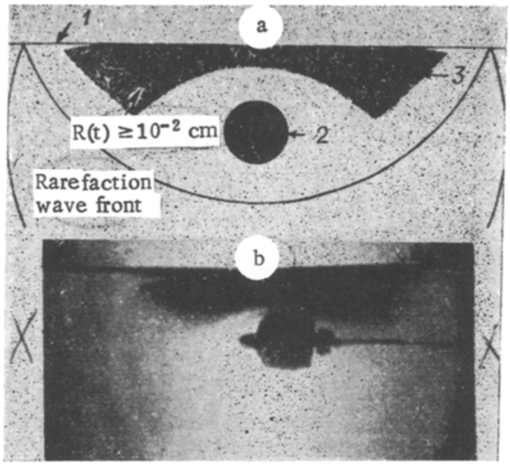

Fig. 1

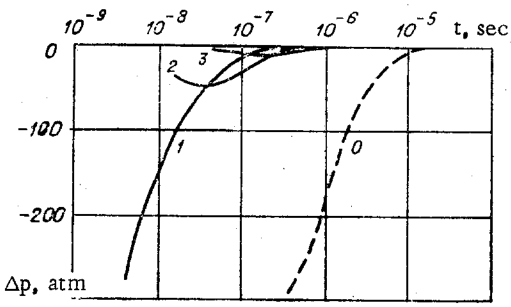

Fig. 2

amplitude is applied instantaneously (curve 1), or for a time during which the cavitation nucleating centers have not expanded significantly (about 10^{-9} sec in our case), then this phenomenon is recorded. However, the time during which the medium manifests it is very small: After 10^{-8} sec, the negative pressure in the rarefaction wave is reduced twofold, and at 10^{-7} sec it has almost completely disappeared. Thus, if the equipment resolving times is within this limit, then when the above condition is fulfilled, the large tensile forces can be recorded experimentally. With a change of the slope of the rarefaction wave front from 0 to 10^{-6} sec (curve 3), the maximum sizes of the tensile stresses tolerated by the real liquid are reduced by two orders of magnitude. Curve 2 is a slope of the front of 0.1 μsec and curve 0 is the case of a one-phase liquid.

As shown in [22], the magnitude of the negative pressures, calculated within the framework of the two-phase medium model, correspond completely with the experimental data. It should be stressed that according to the calculation in [22], the instant of appearance of visible cavitation and the maximum of the negative pressure do not coincide. Thus, for the example given above, with a rise time of 0.1 μsec for the rarefaction wave front, the maximum negative pressure (about 40 atm) originates at $(3\text{-}4)\cdot 10^{-8}$ sec, but the visible cavitation only originates at $(3\text{-}4)\cdot 10^{-6}$ sec.

It is interesting to note that if in a certain region the rarefaction wave front is quite flat (for example, a few microseconds), then visible cavitation in it will be greatly different from the estimate by the one-phase model (the same experimental fact was pointed out above). The reason is determined unambiguously: During the rise time of the front, the concentration of free gas has increased by several orders of magnitude and, although the cavitation bubbles have not reached a visible size, the state of the medium has changed significantly without allowing the development of large stresses.

The difference between the experimental data of Knapp, Davis, and Brown [20] now also becomes understandable. The insignificant magnitudes of the negative pressures obtained by Knapp are determined by the flow rise and relative large duration of the applied stress (the experiment was carried out with a Venturi tube). In the experiments of Davis and Brown, the limiting pressure of the onset of cavitation was measured in a shock tube containing liquid with a free surface. Below there was a piston, which initiated a compression wave as a result of the impact of a lead projectile on it. Davis measured the tensile forces with a tensometer (i.e., to a considerable degree, their integrated characteristic curve), and Brown measured the pressure with a piezosensor. The spread in his data is due to the change in mass of the piston and, consequently, also in the rise time of the rarefaction wave magnitude..

Thus, on the basis of what has been said, two principal features can be recorded: The model of the two-phase medium permits the dynamics of the cavitation zone and the parameters of the rarefaction waves in a real liquid to be investigated; once and for all, there are no established critical values of the tensile stresses causing the development of cavitation (or its breakdown) in a real liquid, even if the gas content of the liquid is fixed.

2. Cupola

Analysis of the results of the investigations of the motion of the free surface of the liquid in the case of underwater explosions obviously must begin with the development of a cupola. The reason for its formation is due to spallation effects during the interaction of the shock wave with the free surface. However, there are two points of view concerning its structure: The cupola is an assembly of layers of a continuous mass of liquid, breaking off along the lines of the cavitation discontinuity in the main mass [8]; the cupola has a drop structure [1]. The second point of view, expressed by Pekeris [1], is more justifiable. In experiments on the explosion of depth charges, he established that the retardation of the growth of the cupola exceeds the acceleration of gravity by a factor of three. Two causes are possible: the effect of air resistance during atomization of the drop (then the cupola is spray) or the action of the external excess pressure on the moving continuous layer of liquid. Pekeris shows [1] that estimates of the cupola thickness according to the continuous layer must amount to not less than 3 m, which is not very probable.

Obviously, the confirmation of either model of the cupola structure in experiments with a concentrated charge is desirable, but it is unrealistic in view of the axial symmetry of the process, which does not permit the cupola to be considered "in section." An experiment in plane formulation, which was conducted when investigating the flow characteristics of the liquid with the free surface in the case of an underwater explosion [23], serves this purpose. An explosive charge was positioned between two parallel plates partially immersed in the liquid and perpendicular to it. The gap between the plates was equal to the length of the charge (2-3 cm). That part of the plate located above the free surface was made of plastic, which permitted the flow structure to be studied by means of high-speed streak photography. Figure 3a, b shows two characteristic streak frames of the development of spallation 1 for charge depths of 10 and 6 cm, respectively, and for a charge radius of a_0 = 3.2 mm. It can be seen that for large charge depths the cupola, in fact consists of a collection of layers (but not continuous) and the cavitating liquid. When the charge depth is reduced, a concentration of the spallation zone is observed close to the axis of symmetry. The shape of the spallation (Fig. 3b) indicates a tendency toward the formation of a cumulative recess at the free surface. The presence of bubbling cavitation in the spalled layers leads to the breakdown of their continuity. Because of this, in the subsequent instants, the spalled mass of liquid collapses and a spray cupola is formed.

We note that the explained case of the structure of the spallation layers once again confirms the necessity for correcting the existing concepts of the mechanism of cupola development in a liquid, since the condition of spallation becomes indeterminate even at the critical pressure and at the site of formation of the discontinuity.

The parameters of the cupola (shape, height, and ascent velocity) have been investigated experimentally for a wide range of explosive charge weights (from tenths of a gram to hundreds of kilograms) and explosion depths (from unity to tens of charge radii) [1, 2, 7, 8, 12]. We shall derive the basic empirical relations for a cupola; the following notation is used: v^* is the initial ascent velocity; Q is the charge weight (in [2], in pounds); h is the explosion depth, m; H is the height of a surface point of the cupola; r_1 is its radial coordinate; B and C are constants which depend on the charge type; t is the time; a superscript 0 denotes a quantity taken relative to the charge radius a_0; a superscript asterisk denotes a parameter of the central point of the cupola; and m and n are constants which depend on h_0.

Reference [2]: charge weight up to 4.5 kg, explosion depth $0.3 < Q^{1/3}/h < 4$,

$$v^* = 66(Q^{1/3}/h - 0.1)\ \text{m/sec},$$

$$H = B\left(\frac{h^2}{h^2 + r_1^2} - C\right).$$

Reference [8]: charge weight 0.2 g, explosion depth $1 \leqslant h_0 \leqslant 17$

$$H_0^* = m \ln(1 + n t_0). \tag{2.1}$$

The coefficients m (dimensionless) and n (m/sec) can be determined by the experimental curves of H_0/t_0 in [8]. According to Eq. (2.1), $v^* = mn$, and on the basis of the data of [8], it can be within the limits of error of up to 10% and is represented by the relation $v^* = 3540 h_0^{-1.3}$ m/sec, which is valid for the range of explosion depths $2 \leqslant h_0 \leqslant 8$.

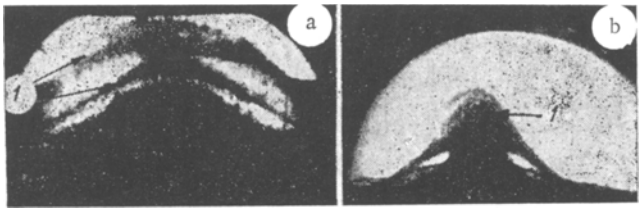

Fig. 3

Reference [12]: charge weight 100 kg, depth of reservoir 12 a_0,

$$v^* \simeq 4500 h_0^{-1.3} \text{ m/sec}, \qquad (2.2)$$
$$H_0^* \simeq K(85 + 49t_0 - 1.3t_0^2).$$

The coefficient K depends on h_0 and has a maximum at $h_0 = 8$. We note that the nature of the change of magnitude of the initial velocity of the central point of the cupola with increase of the explosion depth is identical for relation (2.2) and for the relation obtained above from the data of [8]. Only the coefficients are different, which confirms the anomalous increase of the ascent velocity of the cupola for large scale charges, as stated in [1].

It can be seen that in the region of explosion depths $h_0 \leq 6$–7, the velocity of the cupola will be supersonic. This leads to the onset of intense shock waves in the atmosphere. This fact has been noted and investigated experimental y in [7]. In [9], it was shown that for explosion depths of $h_0 = 1$–3, the amplitude of the shock wave in the atmosphere is described satisfactorily by the well-known relation for supersonic flow of a gas around a blunt body, the effect of the leading edge of which is equivalent to the effect of a cylindrical explosion

$$\Delta p / p_0 = 0.24/\zeta^2 + 0.48/\zeta^{0.75}, \qquad (2.4)$$

where $\zeta = r/\lambda$, r is the distance from the axis to the point of observation, $\lambda = M_1 d$ is the scale of the cylindrical explosion, d is the diameter of the body (cupola), M_1 is the Mach number of the incident flow, and p_0 is the pressure of the undisturbed gas. It was noted in [9] that for a fixed value of h_0, the quantity λ is proportional to the radius of the charge a_0, and, thus, for identical relative distances r/a_0 from the explosion epicenter, the quantity ζ and, consequently, also the pressure in Eq. (2.4) are found to be identical for different charge weights Q. It is verified that this fact coincides with the law of similarity for an underwater explosion: The initial velocity of the cupola depends only on h_0 [7, 12], and its geometrical parameters for h_0 = const and t_0 = const are proportional to a_0.

3. Plumes

In the problem being considered, it is obvious that the greatest interest arises in the directional vortical and radial eruptions (plumes) at the free surface of the liquid. As it will be seen below, according to the nature of the plume development, the explosion depths are divided into two groups (two ranges of values of the parameter h_0): depths at which secondary waves (pulsations), resulting from the collapse of the cavity with the detonation products, are absent, and those for which secondary waves occur. These groups have a quite sharp limit in the region of $h_* = h/a_* \approx 1$, where h is the explosion depth and a_* is the maximum radius of the cavity with the detonation products in the case of an explosion in an unbounded liquid ($a_* \approx 135a_0$ for an infinite cylindrical charge of radius a_0; $a_* \approx 30a_0$ for a spherical charge with a hydrostatic pressure of 1 atm). When describing the external features of development of plumes with change of h_0, we shall follow references [2] and [1], illustrated in Fig. 4, where the natures of the development of surface plumes and the explosion cavity are compared.

Explosion Depth h_0 = 3–4. The first phenomenon observed at the free surface is the development of a cylindrical (in all probability) flat plume, at the center of which at subsequent times there grows a very thin and narrow column of water (see Fig. 4a1). This description, given in [2] for charges with a weight of 4.5 kg, corresponds exactly to the frame-by-frame photography of plume development shown in [12] for the case of explosion of a 100-kg charge at a depth of h_0 = 4.

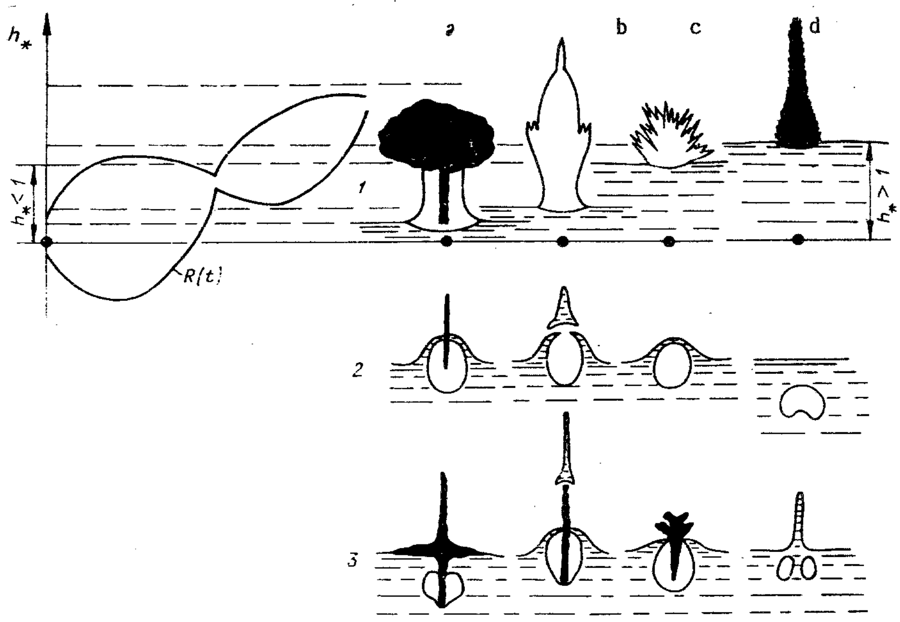

Fig. 4

Explosion Depth $h_0 \simeq 6$. Only the trace (possibly) of a flat cylinder can be seen. For the first time, the appearance of a cupola, from which a thin narrow jet of water is growing, is observed. With increase of the explosion depth, this jet becomes wider and clearly more dense (see Fig. 4b1).

Explosion Depth $h_0 \simeq 12$-13. The jet appears somewhat later, and its maximum height is somewhat less than at the previous depths. With further increase of the explosion depth, the cupola attains its maximum height before the het of water appears above it.

Explosion Depth $h_0 \simeq 50$. The cupola falls before the cavity with the detonation products "tears it into a feather shape" (type of Fig. 4c1).

It can be seen that the description of the plume development given in [2] has started with one group of depths and concluded with another, in which the explosion cavity can perform at least one pulsation. It will be interesting to compare these data with the description given in [1] for the cases of the explosion of charges with weights of 137 and 450 kg.

Explosion Depth $h_0 \simeq 6$. "Eruption of the bubble" occurs during its rapid expansion, and almost immediately "following this" a very narrow vertical fountain appears, reaching a considerable height (about 300 m).

Explosion Depth $h_0 \simeq 20$. The gas bubble reaches the free surface earlier than the start of its compression. As a result of its "eruption," radial fountains originate (see Fig. 4c1). Frame-by-frame photographic data shows that in this case the radial fountains are very similar to the plumes of the "feather structure" described in [2].

In the case of the explosion of the charge of weight 137 kg at a depth of 8 m ($h_* > 1$, the second group of depths), the explosion cavity reaches the surface at the instant of maximum compression, having a high buoyancy velocity. Almost all the water located above the cavity is ejected vertically upward, forming a high narrow fountain [1] (see Fig. 4d1). With an explosion at a depth of 13.7 m, the feather type of eruption structure recurs (see Fig. 4c1), and at 19.8 m, the vertical fountain of type Fig. 4d1 (but of smaller dimensions) [1]. The relation between the external eruption effects and the pulsation stages of the explosion cavity is confirmed by the experimental graph in [1], according to which the points of intersection of the curves of the times of onset of eruption and the pulsation periods correspond to the maxima of the dependence of the rate of development of the eruption on the explosion depth.

Thus, generally speaking, it can be concluded that depending on the explosion depth, three types of plumes occur: a cylindrical cavity with subsequent formation of an internal vertical jet, a vertical cavity, and a radial cavity.

The mechanics of formation of the first type of plume is quite obvious. But it is not possible to say anything about its central jet (we shall confine ourselves here for the present to statements of fact). The authors unanimously designate the cause of the origination of vertical plumes — the rapid first (or second) expansion of the cavity with the detonation products. However, the concept of its structure is different: In [1] this is the "eruption of the explosion bubble," i.e., of the detonation products (the authors of [8, 10], for example, hold the very same opinion), while in [2], this is, "obviously, a dense liquid jet," According to the description of the nature of the eruption in the case of $h_* > 1$ [1], it may be assumed that he has in mind a dense mass of liquid.

As a result of the experimental investigations mentioned above, the plume parameters were determined (or estimates were given): the time and height of ascent, diameter of the neck and base of the plume, and the amount of liquid in the eruption. It should be mentioned that almost all these parameters were investigated for the first group of explosion depths, i.e., in the region $h_* < 1$. If the first four of the parameters listed are only statements of the external effects and are determined simply by high-speed streak photography, then then the estimate of the mass of liquid in the eruption depends significantly on the concept of its structure. These estimates are contained in [8, 10]. According to the first of them, the mass of ejected liquid is determined by the volume of the maximally expanded cavity with the detonation products and is expressed by the relation $M \approx 540QH_1/a_0$, where H_1 is the depth of the reservoir (the dependence is derived for shallow reservoirs). The authors suppose that this is the upper limit of the required value and note that the main mass of water is found to be in the base of the plume. According to this relation, the mass of ejected liquid is independent of the explosion depth, which is very doubtful. In [10], an expression is proposed for the maximum quantity of liquid, ejected to a high altitude, $M_* \approx 150Q$. This figure is obtained by the authors for the range of charge weights of 0.075-136 kg (the method of estimation is now known), but it contradicts their concept of the vertical eruption as a bundle of detonation products with water spurts.

Thus, the question concerning the structure of the directional vertical throwout and the mechanism of its formation remains open. The necessity for explaining this question in relation to the contradictions existing in the literature is obvious.

The first premise, which considerably simplifies the formulation of the experimental investigations in this field, is determined by the results of all the previous reports: This is the similarity of the phenomena in the first stage of the explosion for a wide range of charge weights. The second important fact, observed as a result of laboratory investigations, carried out by the authors at the Institute of Hydromechanics, Siberian Branch Academy of Sciences of the USSR, consists in that the qualitative effect is maintained both in the case of explosion of charges of small weight in the axisymmetrical (lumped charge) and plane (cord charge) formulations, as well as in the case of the electrical explosion of a wire in these same formulations. This considerably widens the possibilities of analysis of the principal facets of the phenomenon.

The investigations showed that in all the cases stated above, just as for explosions of large-scale charges, in the range of the first group of charge depths $0.1 \leq h_* < 1$, in its first stage (first half period) a dense high-velocity vertical jet of liquid is observed above the cupola [Fig. 5a — $Q \approx 1.5$ g, $h = 5$ cm; Fig. 5b — $Q = 10$ kg, $h = 1.1$ m; 1) free surface; 2) jet].

In [23], based on the results of investigations in the plane formulation (see Sec. 2) for the explosion of a long Tretyl charge of radius $a_0 = 3.2$ mm, it is shown that under the spray cupola a jet of uniform liquid is formed almost simultaneously with it, this jet subsequently puncturing the cupola and appearing above it. Its development is caused by the rapid expansion of the cavity with detonation products, which is recorded particularly clearly in experiments with exploding wires: In this case, the shock wave is weak and almost no spallation effects result.

Consequently, the rate of growth of the vertical plume, the time and altitude of its ascent, and the mass of liquid in it at large height will be determined by the parameters of this jet.

By means of a special trap, measurement of the mass of liquid M in the jet in the case of the underwater explosion of a capsule-detonator ($Q \approx 1.5$ g) at different depths were carried out by the author. The trap was a flask with a tapered entrance, the opening of which

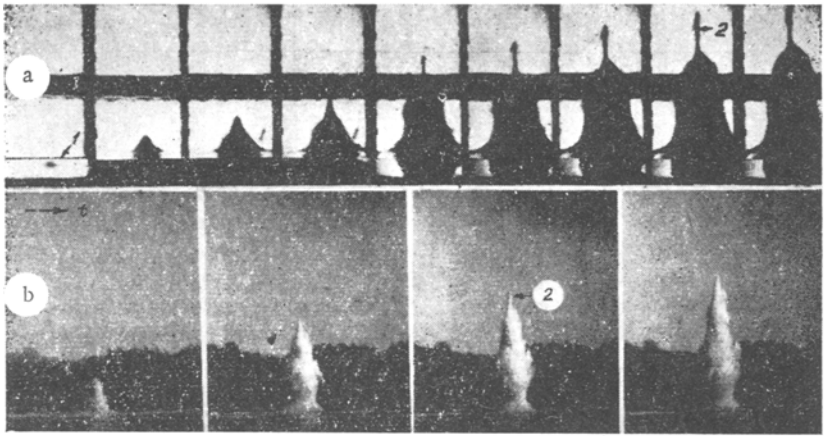

Fig. 5

was aligned with the charge vertically and was positioned at a height exceeding the maximum height of ascent of the cupola. It was found that at an explosion depth of $h_* \simeq 0.6$, the $M(h_*)$ curve has a sharply pronounced maximum ($M_* \simeq 440$ g $\simeq 300Q$) with a significant fall along both sides from the stated depth (Fig. 6, curve M). The data were obtained for the height of the trap of 4.4 m, and for a height of 2.4 m it was found that $M_* \simeq 500$ g (here the cupola reaches the trap). The process of entry of the jet into the trap was recorded on moving-picture film. The experimental curve of $M(h_*)$ over the range of explosion depths $h_* \leq 0.6$ can be approximated by the expression $M \simeq 545Qh_*^{1.4} \simeq 4.63Qh_0^{1.4}$ with an error not exceeding 10%. The value of h_* obtained in the experiments on small charges, corresponding to M_*, coincides with the results of [10], $h_* \simeq Q^{1/3}$ m (Q, kg).

An empirical relation has been found for the maximum velocity of the apex of the jet at the instant of its appearance above the cupola, $v \simeq 13.3h_*^{1.66}$ m/sec (see Fig. 6, curve v), which is valid over the range of explosion depths $0.1 \leq h_* \simeq 1$. The data of [9] on the velocity of ascent of the central point of the cupola for $h_0 = 2$–3 and $Q = 0.2$ g lies almost on the continuation of the $v(h_*)$ curve in the region of $h_* < 0.1$. Hence, it can be seen that at these explosion depths it is not possible to construct a boundary between the development of the cupola and the plume.

Based on the results obtained in [10] and by the present author, below we present the principal experimental characteristics of the jet flow of the vertical eruption, originating in the first stage of an underwater explosion for the first group of depths (Q, kg; h, m); $H_* \simeq 60Q^{1/4}$ m is the maximum height of ascent; $h_1 \simeq 0.5\ Q^{1/4}$ m is the explosion depth corresponding to it; $T_* \simeq 2.25\ Q^{1/8}$ sec is the time of ascent up to H_*; $M_* \simeq 300\ Q$(kg) is the maximum amount of liquid in the jet; $h_2 \simeq Q^{1/3}$ m is the explosion depth corresponding to it; and $v \simeq 29Q^{0.55}h^{-1.66}$ m/sec is the maximum velocity of the jet.

Thus, the parameters of the jet flow also are determined, and it remains to explain the mechanism of its formation, i.e., to determine in what manner; as a result of the explosion cavity close to the free surface, the directional jet flow arises. In order to explain this question, the author carried out numerical investigations in the following formulation [23, 24].

In the half space of an ideal incompressible weightless liquid, occupying a region $Q(t)$, at a depth H below the free surface $\zeta(t)$, a cylindrical cavity $R(t)$ with the detonation products (plane formulation) is expanding under the action of a high pressure. At $\zeta(t)$, the pressure is constant and is equal to the atmospheric pressure p_0, while at $R(t)$ it varies according to a known law. For the potential flow, the formulation of the problem is described as follows:

$$Q(t): \qquad \Delta\varphi = 0, \qquad \varphi \to 0 \quad \text{as} \quad |\mathbf{r}| \to \infty; \tag{3.1}$$

$$\zeta(t): \qquad p = 1, \ \ \varphi_t - (\nabla\varphi)^2/2 = 0; \tag{3.2}$$

$$R(t): \ \ p(t) = p(0)\,[S(t)/S_0]^{-\gamma}, \ \ \varphi_t - (\nabla\varphi)^2/2 = p(t) - 1, \quad \text{for} \quad t = 0; \tag{3.3}$$

$\zeta(0)$ is the horizontal surface, $\varphi = 0$ at $\zeta(0)$; $R(0)$ is the circumference, $\varphi = \text{const}$ at $R(0)$;

$S(0) = S_0 = \pi a_0^2$; and $p(0) \gg 1$. Here $\varphi' = \varphi\sqrt{p_0/\rho_0}\cdot a_0$, $t' = t\sqrt{\rho_0/p_0}\cdot a_0$, $\mathbf{r}' = \mathbf{r}a_0$, $p' = pp_0$, $H' = ha_0$; and the primes denote dimensional quantities. The problem is solved by a combination of two methods: the Euler hydrodynamic approximation (EHDA) for solution of the Laplace equation (3.1) and a differencing method for solving Eqs. (3.2) and (3.3). The procedure for the calculation consists in the following.

If, at the moment in time t_i, a distribution φ is given at the known boundaries $\zeta(t_i)$ and $R(t_i)$, then on electrically conducting paper over the contour of the boundary of the region $Q(t_i)$, its electrical analog is established by means of a special busbar, and the potential gradient is found. Using the scale factor for conversion to hydrodynamic quantities (determined for each moment in time), according to the values found and the directions of the mass velocities for the next moment in time the new boundaries of ζ and R are plotted, and new values of the potentials on them are determined by the difference analogs of the Cauchy—Lagrange integrals [Eq. (3.2) and (3.3)]. The stability of the solution is verified experimentally by the choice of step with respect to r and t. Since, during the calculation, the boundaries are plotted with respect to one and the same control point, at which the potential distribution is given, the kinematic condition for the equation of the boundary $q(\mathbf{r}, t) = 0$ of the region $Q(t)$ is fulfilled automatically.

The results of the flow calculation for the case $h_0 = 4$, $p\ 0 = 4\cdot10^4$, and $\gamma = 3$ [the shapes of the free surface $\zeta(t)$ and the cavity $R(t)$, equivalent lines and streamlines] at different instants are shown in Fig. 7a (the figures correspond to times 0, 2.7, 5.8, 8.7, 127, 208, and 408 μsec). In Fig. 7b the calculated mass-velocity curves v(t) are given for three points on the free surface I-III and on the surface of the cavity 1-3. Analysis of the results of the calculation showed that the liquid above the cavity with the detonation products, produced at the initial moment in time, vertically above the pulse with maximum velocity along the axis of symmetry, at subsequent moments in time is moving by inertia and is pulled out into a jet. It can be seen in Fig. 7b that up to approximately 20 μsec the nature of motion of all points is identical. At approximately 5-6 μsec, the acceleration of the liquid "particles" changes its sign at the wall of the cavity: The velocity starts to decrease. The velocities of the liquid "particles" at the free surface at first "emulate" behind the behavior of the gas cavity boundary, and then the inertial forces start to predominate and the particle velocity increases. For a particle on the axis of symmetry (curve I) at $t >$ 50 μsec, it is as if the gas cavity no longer exists, and its velocity increases rapidly. At these instants, the jet starts to form (Fig. 7a). Points which are more remote from the axis (curves II and III), shortly after start to emulate behind the behavior of the cavity (their velocity decreases), the effect of which is found to be significant for them. The growth of the jet has a marked effect on the expansion of the upper half of the cavity, and the velocity of the particles decreases markedly more slowly by comparison with the lower part of the surface.

The determination of the mechanism of formation of the jet flow has permitted a simple and graphic model of the phenomenon [24] to be proposed. In fact, in order to impart the required momentum to the layer of liquid, it is not essential to use an explosive process: It is sufficient to throw a solid sphere or a cylinder, initially immersed in the liquid, in the direction of the free surface. The result of this experiment with a sphere is shown in Fig. 8 and the calculation in the plane formulation is given in [24]. In the concluding stage of motion, the sphere was braked in a special way in order to simulate the process of development of the explosion cavity. Thus, the layer of liquid remaining on the surface of the sphere after stopping (at this moment in time, one half of the sphere was above the free surface), gains the possibility of moving by inertia under the action of the acquired momentum. It is easily seen that in this case a jet is formed.

When analyzing the spalling phenomena, it was shown by the fact of concentration of the spallation zone in the vicinity of the axis of symmetry that the shape of the spallation provides the basis for confirming the presence of a cumulation recess at the free surface. This phenomenon was first predicted by M. A. Lavrent'ev; based on it, he proposed a model of the formation of the vertical jet at the free surface [14]. The essence of this consists in the following. As a result of spalling, originating by the interaction of the shock wave from an underwater explosion with the free surface, a recess is formed in the latter. The expanding cavity with the detonation products, in which approximately one half of the energy of the explosion is contained, creates a velocity field which is orthogonal to the surface of the recess. Inflow of the recess leads to the formation of a cumulation jet. As shown above, a directional vertical jet develops also on the initially plane surface. However, the pres-

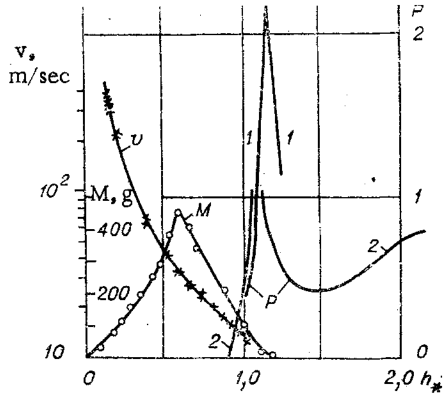

Fig. 6

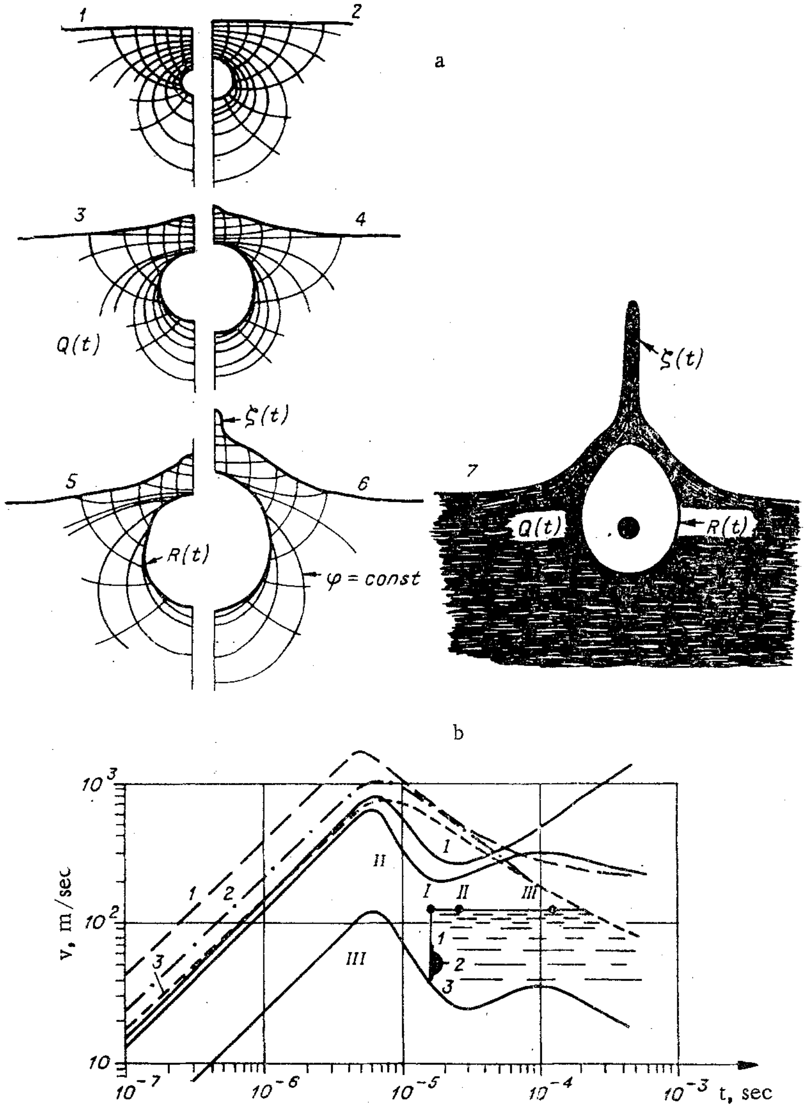

Fig. 7

ence of the recess creates sharper mass-velocity gradients along both sides from the axis of symmetry, which must lead to velocity gradients along both sides from the axis of symmetry, which must lead to a significant intensification of the effect. This is confirmed by the results of a calculation carried out in a formulation similar to that discussed above (with similar parameters). Figure 9 shows the shape of the boundaries at the moments in time t = 0-10 μsec. It can be seen that the jet formed from the recess achieves the same parameters as in the case of the plane surface, after a considerably shorter time. On the basis of the data given, the practicality of this model may be assumed to be established.

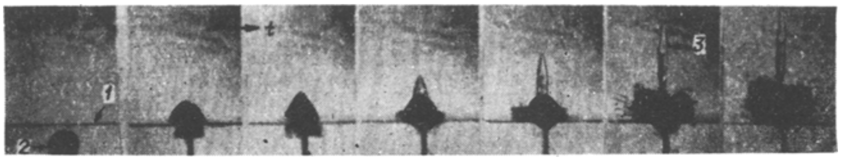

Fig. 8

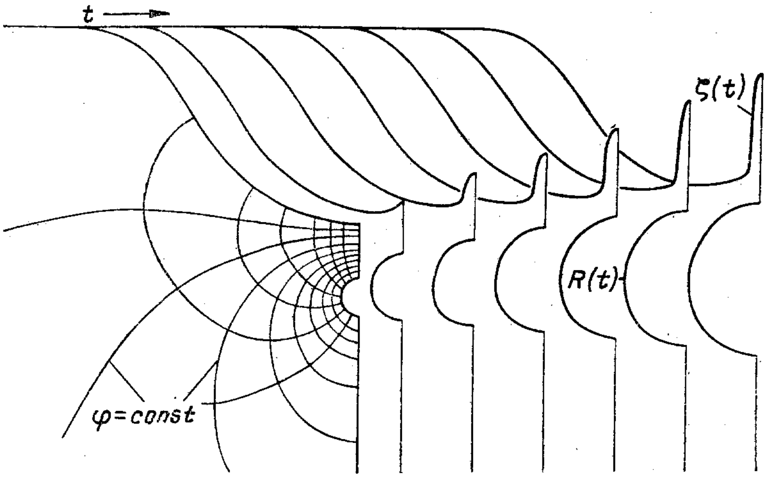

Fig. 9

This result permits the following explanation to be proposed for the anomalous increase of velocity of the central point of the free surface (previously called the cupola), which is characteristic for large charges. According to Eq. (1.3), the increase of the radius of the charge for a fixed relative explosion depth h_0 leads to a change of nature of the development of cavitation near the free surface and, consequently, of the spalling. It is natural to suppose that when large charges are exploded, a more optimum jet is achieved from the points of view of the rate of formation and of the shape of the recess and that the velocity shock of the free surface in the vicinity of the axis of symmetry for small values of h_0 is determined not by the development of the cupola, but by the velocity of the vertical jet, which, as shown above, is formed beneath the cupola almost simultaneously with it.

As the experimental investigations carried out by the author showed, the structure of the vertical eruption for explosion depths of $h_* < 1$ is not confined to the jet flow considered above. Actually, for explosions at these depths, the first pulsation is absent, which is a consequence of depressurization of the cavity with the detonation products — the formation of an open pocket. The moving-picture photographs of the flow-development process at the free surface, photographed in a plane channel with a width of 1 mm, showed that very soon after its formation, the open pocket closes with the formationof a vertical (upward) and an inverted jet (Fig. 10a). It is precisely this vertical jet, originating on the background of a disappearing hollow cylindrical eruption, which is described in [2] and which is clearly seen in the moving-picture photographs of [12]. The figures denote the following: 1) free surface; 2) charge; 3) vertical and inverted jets; 4) pocket; 5) jet.

We note that the outward appearance of the result of closure of the pocket is also the experimental fact of the collapse of the neck of the plume, investigated in [8]. The relative magnitude of the pulsation period of the cavity in the unbounded liquid amounts to $T_0 = T/a_0 \approx 5.5$ sec/m. According to the data of [8], the collapse of the neck depending on the explosion depth starts in the range $t_0 \approx 1$–3 sec/m, i.e., for small explosion depths this process is developed even at an early stage of the first expansion of the cavity with the detonation products. In Fig. 4a2 and Fig. 4a3, respectively, two characteristic successive frames of the moving-picture photographs of the process are shown diagrammatically: the initial stage of jet formation with closure of the pocket and the final stage of collapse of the neck of the plume.

Thus, the basis of the structure of the vertical eruption for the first group of explosion depths (see Fig. 4b1) is the jet tandem (see Fig. 4b2, 3), the first jet of which is formed as a result of the inertial motion of the layer of liquid above the explosion cavity

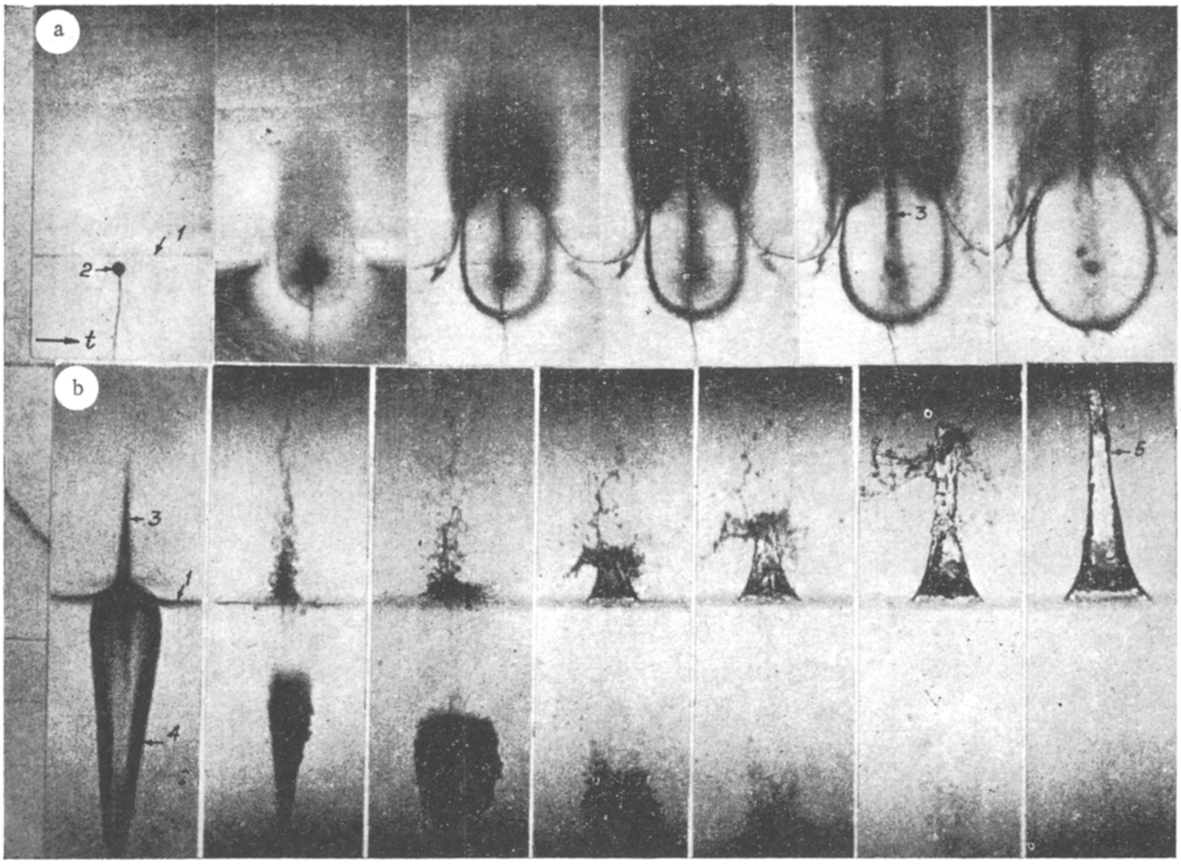

Fig. 10

under the action of the momentum received (see Fig. 4b2) and the second, as the result of closure of the open pocket, formed after depressurization of the explosion cavity (Fig. 4b3). Investigations carried out at the Institute of Hydrodynamics Siberian Branch, Academy of Sciences of the USSR showed that the development of the second jet of the vertical eruptions is completely similar to the nature of the flow originating as the result of surface closure of the pocket by the high-velocity penetration of a bullet into a liquid (see Fig. 10b, First frame). In addition, it can be seen from the data of frame-by-frame photography that after closure of the pocket, its further expansion is observed as a result of the subsequent motion of the bullet and the inertial motion of the liquid. The pressure inside the cavity becomes less than the hydrostatic pressure, which leads to its collapse and recompression. Then the cavity expands and, by analogy with the first stage of an underwater explosion, causes the development of a dense vertical jet at the free surface of the liquid (see Fig. 10b).

At explosion depths approaching $h_* = 1$, depressurization of the cavity with the detonation products takes place in the later stages of its expansion, as a result of rupture of the thin shell of liquid in the region of the axis of symmetry (see Fig. 4c2). The pressure in the detonation products at the moments in time preceding rupture may be significantly lower than hydrostatic. Closure of the pocket formed after depressurization of this cavity leads to the developmentof an intense inverted jet flow and radial eruptions (of the "feather" type) at the free surface of the liquid (see Fig. 4c3).

As has been shown, in large-scale explosions, vertical eruptions are also observed for the second group of depths (depressurization of the cavity with its first expansion does not occur — a pressure pulsation is recorded). Two of these eruptions [1] have been recorded; the start of their development corresponds to the times of the first and second maximum compressions of the explosion cavity, with the condition that during the pulsations the cavity is able to float up quite close to the free surface. At first sight, it can be supposed that the mechanism of formation of these eruptions corresponds completely to what has been said for the first stage of the explosion. However, it should not be forgotten that the phenomenon described refers to the explosion of charges with a weight of 137 kg, of which the maximum radius of the explosion cavity has a size of order 10 m. This means that at the stage of collapse of this cavity, the force of gravity plays a decisive role in its deformation. According to two-dimensional calculations [11], carried out for the "Wigwam" nuclear explosion, in the lower region of the collapsing cavity with the explosion products, a cumulation jet, directed toward the free surface, is formed. Thus, as the final result, the model in [15], suggested on the basis of the results of the solution in the exact formulation of the problem concerning the buoyancy of the bubble, is found to be valid for the vertical eruption corresponding to the first compression of the cavity. According to this model, with a suitable depth of immersion and charge weight, during flotation and deformation of the explosion bubble formed in its lower part, a cumulation jet is formed at the instant of emergence of the bubble at the free surface and it determines the structure of the vertical eruption (see Fig. 4d1-3).

The mechanics of the process of development of a third vertical eruption is not quite so obvious, since at the stage of the second pulsation period, initially the spherical cavity with the detonation products has the shape of a torus. It is natural to expect that at very small distances to the free surface, expansion of the toroidal cavity can lead to the development of a ring-shaped plume. But in this case, is the vertical eruption restricted only to a ring-shaped structure? In order to clarify this question, experimental investigations of the development of the surface effects from the underwater explosion of a ring-shaped explosive charge, the plane of which was arranged parallel to the free surface were undertaken, It was found that in addition to the ring-shaped plume (Fig. 11a), the development of a jet flow along the axis of the charge was observed (Fig. 11b). This complex flow is characteristic only for small explosion depths; with $h_* \geq 0.5$, the ring-shaped plume is almost absent, and the central jet comprises the base of the eruption. Its development is a consequence of the cumulation flow of liquid in the region of the axis of symmetry, as a result of expansion of the toroidal cavity with the detonation products. The presence in it of a symmetrical jet flow, directed into the depth of the liquid, is recorded on the frame-by-frame photographs by a characteristic trace in the zone of cavitation. The latter is developed in the inner region of the torus by convergence of the rarefaction waves originating by the reflection of the focused shock wave from the surface of the toroidal gas cavity. At this, the analysis of the structure and mechanics of formation of directional vertical plumes can be assumed to be completed.

4. Anomalous Effect of the Secondary Pressure Field

As the experimental investigations carried out at the Institute of Hydrodynamics, Siberian Branch, Academy of Sciences of the USSR showed, radial plumes are of a double origin. One of them is due to closure of the pocket, while the other is due to the special features of the secondary pressure field; we shall pass on to its analysis below.

In [1], the existence of an anomalous effect of the increase of amplitude of the first pressure pulsation, which is characteristic for a narrow range of explosion depths in the region of $h_* \approx 1$, was shown for the first time. It was recorded experimentally for charges with weights of 137 kg, 250 g [1], and 1.5 g. The curves of the maximum amplitude of the first pulsation P, plotted relative to its value for an unbounded liquid versus the explosion depth, are shown in Fig. 6 (curve 1, 250 g [1]; curve 2, 1.5 g). The sharp increase of the pulsation amplitude does not fall within the bounds of the well-known estimates [1], the models of which, while taking account of the effect of the boundary surfaces, assume that the explosion cavity retains a spherical shape. There is Kirkwood's hypothesis [1], which states that when the explosion cavity is depressurized, mixing of the detonation products and the atmospheric air causes certain auxiliary reactions to occur with the release of energy, which is used for the further motion of the bubble. The contradiction in this assumption is obvious.

In order to explain the cause of this effect within the framework of the model formulation with exploding wires, experimental investigations were conducted with optical recording of the deformation of the cavity during its collapse in the vicinity of the free surface and of the compression waves originating at the instant of the cavity reaching its minimum dimensions [23]. These results were compared with the corresponding amplitude values of the first pulsation for the explosion of a charge with weight ~1.5 g. It is shown that in the range of relative explosion depths $h_* \approx 2$ to 1, the cavity acquires the shape of an oblate ellipsoid in proportion to the collapse. For these depths, some reduction of the amplitudes of the first pulsation is characteristic (see Fig. 6, P). For $h_* \approx 1$, collapse of the cavity is accompanied by the formation of a cumulation jet, directed from the free surface. For $h_* < 1$,

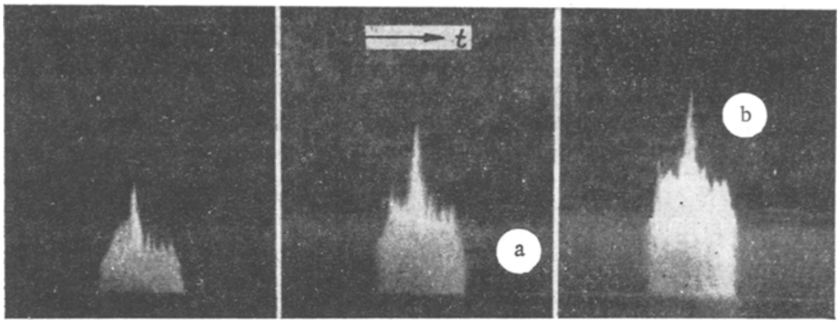

Fig. 11

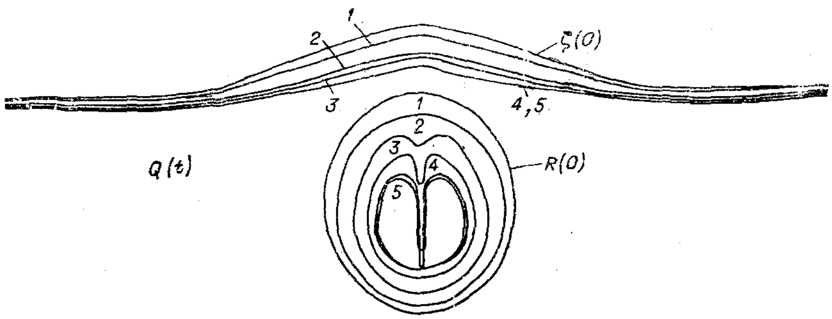

Fig. 12

the cavity is depressurized and closure of the pocket and also the inverted streamer and collapse of the neck of the plume are observed (see Fig. 4a3). The bubble is found to be joined with the free surface as the so-called tubular region, and the inverted streamer has separated. In this case, the first pulsation and the wave pattern disappear completely. Thus, the anomalous effect can be explained only by the penetration of the high-velocity cumulation jet into the liquid.

In the plane formulation, within the framework of an ideal, incompressible, weightless liquid, using the EHDA method, a calculation has been carried out on the deformation of an empty cavity during its collapse close to a free surface (Fig. 12). The initial shapes of the free surface $\zeta(0)$ and of the cavity R(0) were chosen from experiment, and the initial mass velocity was assumed to be zero. The results of the calculation are given for times 10^{-2}, $1.295 \cdot 10^{-2}$, $1.596 \cdot 10^{-2}$, $1.7 \cdot 10^{-2}$, $1.776 \cdot 10^{-2}$, and $1.782 \cdot 10^{-2}$ sec ($h_* = h/a_* = 1$). The calculation has shown that during collapse of the cavity in the vicinity of the free surface, a cumulation jet, the apex of which has a velocity of 300 m/sec (with an average velocity of the wall of the cavity of about 20 m/sec) at the instant of reaching the lower boundary of the cavity, is formed. This result corresponds to a time of about $18 \cdot 10^{-3}$ sec, while the time of collapse of this cylindrical cavity in an unbounded liquid amounts to approximately $26 \cdot 10^{-3}$ sec. Further collapse of the cavity in this interval of time leads to a rapid increase in the velocity of its boundaries, and, therefore, it is natural to expect also a significant increase in the velocity of the cumulation jet. We note that its maximum should occur at a time which is close to the time of collapse.

Thus, at the end of the first pulsation period, for $h_* \approx 1$ the cavity is found to be divided into two parts (in the plane case) by the cumulation jet or is transformed into a torus (into an axisymmetrical torus) which collapses because of the inertia of the liquid to pressures which considerably exceed the hydrostatic pressure. As a result of their subsequent expansion under the free surface, the shape of which in the initial stage of the second pulsation is significantly different from plane, the development of radial jets is observed [24]. The result of the calculation agrees completely with the experimental facts. It should be taken into account that in the real situation before the start of the process of collapse, a certain momentum is imparted to the layer of liquid above the cavity during its first expansion. Because of the inertia, part of the liquid of this layer continues to move upward even during collapse of the cavity, forming a slowly developing vertical jet. The radial plumes are superimposed on this flow, creating the impression of near-simultaneity of the process.

The analysis of the results of experimental and numerical investigations of surface effects given in this paper permits two principal conclusions to be drawn: 1) The jet flows form the basis of the flow structure with the free surface in the case of an underwater explosion; 2) the use of the mathematical model of a two-phase medium in order to describe the wave processes in the actual liquid is promising.

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INTERACTION OF SHOCK WAVES FROM THE SUCCESSIVE UNDERWATER EXPLOSION OF SPHERICAL CHARGES

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§1. An experiment was carried out in an unbounded liquid according to the scheme shown in Fig. 1a-c (d is the distance between the centers of two identical spherical charges 1 and 2 and the initiation of the second charge can be effected with a specified time delay τ). Typical records of the shock-wave interaction corresponding to the cases a-c of Fig. 1 are shown in Fig. 1d-f.

In the case $\tau = 0$ (Fig. 1a, d), symmetrical interaction of the spherical shock waves occurs in the plane of symmetry OO_1, perpendicular to the axis of the system of charges aa_1. It is well known that when a spherical shock wave is incident on a plane at an angle greater than a certain critical angle, irregular reflection of the shock wave, accompanied by the formation of a Mach configuration occurs; CC_1C_2 is the region of its propagation. It should be noted that this case was considered in [1], where the process of the nonlinear interaction of spherical shock waves in water was investigated experimentally and numerically up to the instant of formation of the Mach wave.

When $\tau > 0$ (see Fig. 1b, e), two types of spherical shock-wave interaction occur. For $\tau < \tau_0 = d/D_1$, where D_1 is the velocity of the first shock front, the region of their nonlinear interaction is located between two curvilinear surfaces, formed by the rotation of the lines CC_1 and CC_2 around the axis aa_1. With increase of τ, the angles of inclination of the lines CC_1 and CC_2 to the axis are decreased, and, finally, when $\tau \geq \tau_0$, the explosion of charge 2 occurs in the region behind the shock front 1 (Fig. 1c, f), and the shock wave from the explosion of charge 2 is propagated with a high velocity, overtaking shock wave 1 (overtaking process). We shall call the point C at which the front of the second shock wave overtakes shock front 1 the overtaking point. The region of propagation of the resulting shock wave (RSW) is bounded by the surface C_1CC_2, which is symmetrical relative to the axis aa_1, and the surface area of the RSW front increases with time.

The experiment was conducted with a system of two charges (each with a weight of 2 g) in a water tank 1.5 m × 1.8 m × 2 m having transparent windows. Photography of the process was carried out by means of an SFR-1 high-speed moving-picture camera in a shadow facility with pulsed illumination. The frames clearly show that in addition to the interaction of the shock waves described above, after each of the shock waves s strikes the explosion bubble of the opposite charge, rarefaction waves r are formed. In the region of intersection of the rarefaction waves, a cavitation zone b is formed. It is interesting to note that in the experiments being considered, a Mach reflection of the rarefaction waves M was observed.

This paper is devoted to the investigation of the interaction of shock waves for $\tau > \tau_0$: the overtaking process and the parameters of the propagating RSW.

§2. In order to carry out and investigate the successive underwater explosion of two spherical charges, the following procedure was used. In an open reservoir of depth 6 m two identical cast spherical charges of Trityl/Hexogen 50/50 were positioned at a distance of 3 m from the free surface. The weight of each charge Q = 100 g, radius $r_0 = 2.5$ cm, and the distance between the centers of the charges was varied over the range $d = (2\text{–}30)r_0$. The axis passing between the charge centers is parallel to the free surface. Two piezoelectric pressure sensors for recording the shock waves were positioned along the axis of symmetry of the charges on the far side of charge 2 (Fig. 2). The distance from charge 2 to the sensor S1

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